Thomas Tsoi
(02592751/PHI/1)

# Expansion of the universe

## Problem

Denoting the size of the universe at time $t$ to be $x(t)$, the expansion of the universe is described by Einstein's equation of motion:

$$\frac{d^2x}{dt^2} = -\frac{\Omega_M / 2}{x^2} + \Omega_\Lambda x$$

Here the first term of the equation describes the gravitational attraction and the second term describes the vacuum repulsion. Our task is to describe the future expansion of the universe and trace backward to find the age of it under different values of $\Omega_M$ and $\Omega_\Lambda$.

## Methodology

Spreadsheet is used for calculation and plotting the graphs.

Choosing units of length and time such that $x(t)=1$ and $v(t)=1$ at the present time where $t=0$, $x(t)$, $v(t)$ and $a(t)$ are calculated repeatedly using a time interval ($\Delta t$) of 0.1 unit.

For $t<0$,

$$x(n+1) = x(n) + v(n)\Delta t$$

$$x(n) = x(n+1) - v(n)\Delta t$$

$$= x(n+1) - v(n+1)\Delta t + a(n)\Delta t^2$$

Since $\Delta t$ is small, $a(n) \approx a(n+1)$

$$\therefore \quad x(n) \approx x(n+1) - v(n+1)\Delta t + a(n+1)\Delta t^2$$

Although the term $a(n+1)\Delta t^2$ is negligible, it can be easily included in the calculation, hence we have still included it for more accurate results.

## Results

Case 1: $W_{M}=\mathbf{0.75}$, $W_L=0.25$

The universe will expand forever. The age of the universe is $\left(0.7 \times 15 \times 10^9\right) = 10.5$ Gyr old. *(Graph 1)*

Case 2: $W_{M}=\mathbf{2}$, $W_L=0$

Instead of expanding forever, the universe will eventually collapse at $t=2.5$, i.e. $3.75 \times 10^{10} = 37.5$ Gyr from now. Its present age is estimated to be $\left(0.5 \times 15 \times 10^9\right) = 7.5$ Gyr old. *(Graph 2)*

## Comments and Discussions

The two cases we have described, with parameters of $W_M$=**0.75**, $W_L$=0.25 and $W_M$=**2**, $W_L$=0, both estimate the age of the universe to be less than 11Gyr old, which is in fact in conflict with the current estimate of the age of the oldest globules observed, which are about 12Gyr old, and that of the oldest white dwarfs observed, which are about 12.7Gyr old. After all, the age of stars cannot be larger than that of the universe.

Recent observations give an estimate of $W_M$=**0.37**, $W_L$=0.69 for the parameters, which says that the universe is about 13.5Gyr old. This is more reasonable and compatible with the above mentioned observations.

In addition, Einstein's gravitational field equations give that

$$1 = \Omega_M + \Omega_\Lambda + \Omega_k$$

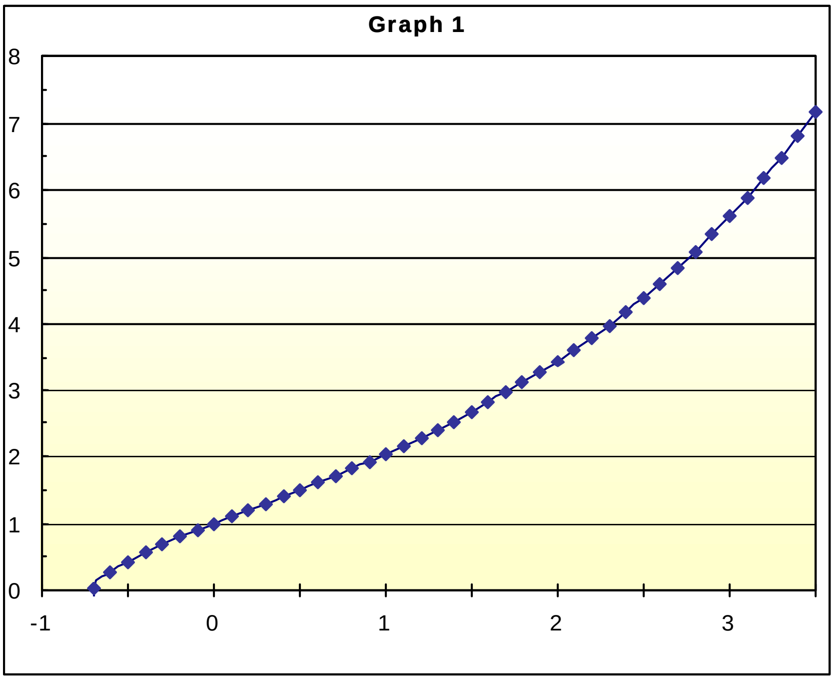
$\Omega_k$ represents the spatial curvature of the universe, i.e. open, flat and closed when $\Omega_k$ is positive, zero and negative respectively. (i.e. case 1 is flat and case 2 is closed.) For $W_M$=**0.37**, $W_L$=0.69, then $W_k$»**0**, that means our universe should be a flat universe. This is compatible with the inflationary model of the universe, which in fact *requires* the universe to be flat.

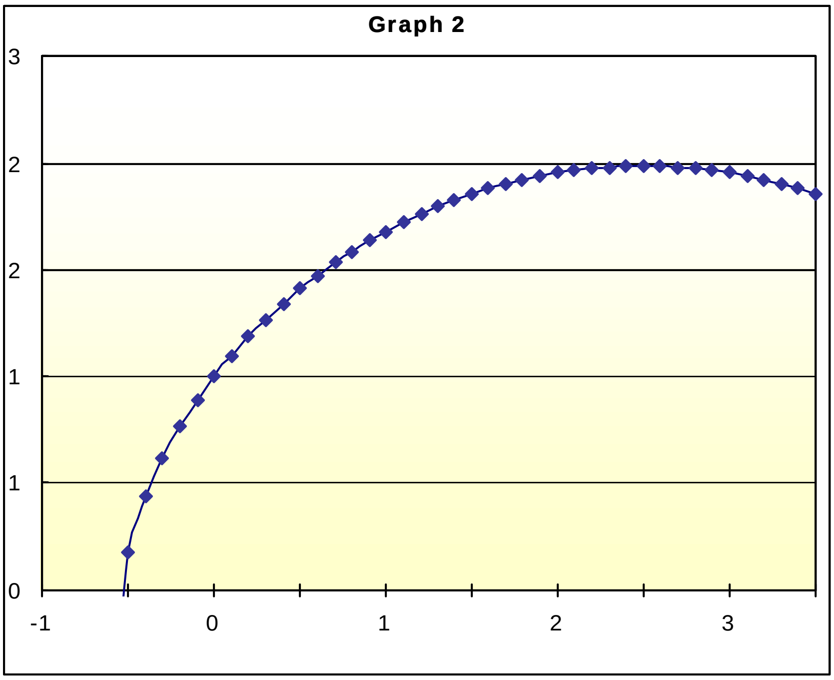
Summarizing the above, our universe is flat in its spatial curvature and has an age of about 13.5Gyr old. It is increasingly accelerating due to the repulsive vacuum force, and thus will expand forever. This model of the universe if now adopted by many cosmologists; nevertheless, there are still unsolved problems associated with it. Hopefully we will see these problems solved soon and new questions come up, after all, that is the approach in which science brings us challenges and knowledge.

---

***Bibliography:***


- Edward L. Wright , *"Ned Wright's Cosmology Tutorial"*; http://www.astro.ucla.edu/~wright/cosmolog.htm
- Edwin L. Turner, *"The Cosmological Constant"*; http://nedwww.ipac.caltech.edu/level5/Carroll/Carroll_contents.html
- Michael R. Feltz, *"Hubble Parameter, Cosmological Constant, Omega"*; http://www.bright.net/~mrf/App9.html


Graph 1
8
7
6
5
4
3
2
1
0
-1
0
1
2
3


Graph 2
3
2
2
1
1
0
-1
0
1
2
3